UNCLASSIFIED

AD 409 091

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA

UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

N-63-4-2

FTD-TT- 63-182

409 091

CATALOGED BY DDC
AS AD No. 409091

TRANSLATION

ON THE CONSTANCY OF TANGENTIAL RUPTURE IN A RAREFIED PLASMA

BY

L. D. Pichakhchi

409091

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

WRIGHT-PATTERSON AIR FORCE BASE
OHIO

DDC
RECEIVED
JUL 22 1963
TISIA D

FTD-TT- 63-182/1+2

UNEDITED ROUGH DRAFT TRANSLATION

ON THE CONSTANCY OF TANGENTIAL RUPTURE IN A RAREFIED PLASMA

BY: L. D. Pichakhchi

English Pages: 8

SOURCE: Ukrainian periodical, Ukrains'kiy Fizichniy Zhurnal, Vol. 6, Nr. 1, January-February 1961, pp 86-92

S/185-61-6-1

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

FTD-TT- 63-182/1+2 Date 28 May 19 63

B

ON THE CONSTANCY OF TANGENTIAL RUPTURE IN A RAREFIED PLASMA

By

L. D. Pichakhchi

On the basis of magnetohydrodynamic equations of Chu, Gol'dberg and Lou have been analyzed problesm concerning the constancy of tangential rupture in a plasma. In general, the case of arbitrary parameters, which characterize the rupture obtained an adequate constancy condition, and for a special case, when the jump is experienced only by the speed of the plasma, a zone of constancy was attained.

We like to point out that the plasma is described by a system of magnetohydrodynamic equations by Chu, Gol'dberg and Lou (1-3):

$$\rho\frac{\partial v_i}{\partial t}+\rho v_k\frac{\partial v_i}{\partial x_k}=-\frac{\partial p_{ik}}{\partial x_k}-\frac{1}{4\pi}H_k\frac{\partial H_k}{\partial x_i}+\frac{1}{4\pi}H_k\frac{\partial H_i}{dx_k}, \qquad (1)$$

where

$$p_{ik}=p_{\perp}\delta_{ik}+\frac{p_{\parallel}-p_{\perp}}{H^2}H_iH_k, \qquad (2)$$

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\rho\mathbf{v}=0, \qquad (3)$$

$$\frac{\partial\mathbf{H}}{\partial t}=\operatorname{rot}[\mathbf{vH}],\quad \operatorname{div}\mathbf{H}=0,\quad E=-\frac{1}{c}[\mathbf{vH}], \qquad (4)$$

$$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho H}\right)=0,\quad \frac{d}{dt}\left(\frac{p_{\parallel}H^2}{\rho^3}\right)=0. \qquad (5)$$

Within the frames of the system of equations (1) - (5) the density tensor of the pulse flow and the density vector of the energy flow have the appearance of:

$$\Pi_{ik}=\rho v_iv_k+p_{ik}-\frac{1}{4\pi}\left(H_iH_k-\frac{1}{2}\delta_{ik}H^2\right), \qquad (6)$$

$$\mathbf{Q}=\rho\mathbf{v}\left(\frac{v^2}{2}+\varepsilon+\frac{p_{\perp}}{\rho}\right)+\frac{p_{\parallel}-p_{\perp}}{H^2}\mathbf{H}(\mathbf{vH})+\frac{1}{4\pi}[\mathbf{H}[\mathbf{vH}]], \qquad (7)$$

where $\varepsilon=\frac{1}{\rho}\left(p_{\perp}+\frac{1}{2}p_{\parallel}\right)$ - sought for internal energy of the plasma. Having expressions (6) and (7), by the known method (4,5) we obtain easily the conditions for tangential rupture:

$$v_x=0,\quad H_x=0,$$
$$\left\{p_{\perp}+\frac{H^2}{8\pi}\right\}=0.$$

The magnetic field H and speed v lie in the rupture area and experience arbitrary jumps. Density jumps ρ and pressure anisotropy $a = p_{\parallel} - p_{\perp}$ are also arbitrary, and the pressure $p_{\perp}$ is connected with the magnetic field by the relationship (9).

The investigation of constancy of tangential rupture will be conducted by a method, which was utilized in[4] in case of magnetic hydrodynamics with scalar pressure.

At small disturbances of the stationary tangential rupture in the case of arbitrary H, v, ρ and also as in[4] it is possible to obtain such conditions for excitations on the surface of separation:

$$v'_x - \frac{\partial\xi}{\partial t} - v_y\frac{\partial\xi}{\partial y} - v_z\frac{\partial\xi}{\partial z} = 0,$$
$$h_x - H_y\frac{\partial\xi}{\partial y} - H_z\frac{\partial\xi}{\partial z} = 0, \qquad (10)$$
$$\left\{p' + \frac{1}{4\pi}(H_y h_y + H_z h_z)\right\} = 0,$$

where v', h', p' - small excitations of values v, H, $p \equiv p_{\perp}$, whereby a system of xyz coordinates was derived, bound with the surface of the stationary separation with axis x directed along the normal to the surface; xi - small displacement value of the surface of separation along axis x; the figurative little bows here and further on designate the difference of the closed in them value on both sides of the separation surface.

We linearize the system of equations (1) - (5). To solve

$$\exp i(\mathbf{kr} - \omega t) \qquad (11)$$

we obtain such a system of linear equations:

$$\omega_0 \mathbf{v}' - \frac{1}{\rho}(p' + \rho\mathbf{u}\mathbf{u}')\mathbf{k} + (\mathbf{k}_0\mathbf{u})\left(1 - \frac{a}{\rho u^2}\right)\mathbf{u}' - \frac{a}{\rho u^2}(\mathbf{k}_0\mathbf{u})\left(\frac{a'}{a} - 2\frac{\mathbf{u}\mathbf{u}'}{u^2}\right)\mathbf{u} = 0, \qquad (12)$$

$$(\mathbf{k}\mathbf{v}') = \omega_0\frac{\rho'}{\rho}, \qquad (13)$$

$$(\mathbf{k}\mathbf{u}') = 0, \qquad (14)$$

$$\omega_0\mathbf{u}' + (\mathbf{k}_0\mathbf{u})\mathbf{v}' - (\mathbf{k}\mathbf{v}')\mathbf{u} = 0, \qquad (15)$$

$$\omega_0\left(\frac{p'}{p} - \frac{\rho'}{\rho} - \frac{\mathbf{u}\cdot\mathbf{u}'}{u^2}\right) = 0, \qquad (16)$$

$$\omega_0\left(\frac{a' + p'}{a + p} + 2\frac{\mathbf{u}\cdot\mathbf{u}'}{u^2} - 3\frac{\rho'}{\rho}\right) = 0. \qquad (17)$$

Here are introduced designations $\omega_0 = \omega - (k_0 v)$, $u = \frac{H}{\sqrt{4\pi}}$ u', a' -small excitations of values u, a; $k_0(0, k_y, k_z)$-true instantaneous vector at the plane of separation. In writing equations (12)-(17) are included conditions (8).

Eliminating solutions $\omega_0 = 0$, which offer true values and can therefore not lead to inconstancy, and eliminating v', we will obtain:

$$\left[(\mathbf{k}_0\mathbf{u})\left(1-\frac{a}{\rho u^2}\right)-\frac{\omega_0^2}{(\mathbf{k}_0\mathbf{u})}\right]\mathbf{u}'-\frac{1}{\rho}(p'+\rho\mathbf{u}\mathbf{u}')\mathbf{k}-$$
$$-\left[\frac{a}{\rho u^2}(\mathbf{k}_0\mathbf{u})\left(\frac{a'}{a}-2\frac{\mathbf{u}\mathbf{u}'}{u^2}\right)-\frac{\omega_0^2}{(\mathbf{k}_0\mathbf{u})}\frac{\rho'}{\rho}\right]\mathbf{u}=0, \quad (18)$$
$$(\mathbf{k}\mathbf{u}')=0, \quad (19)$$
$$\frac{p'}{p}-\frac{\rho'}{\rho}-\frac{\mathbf{u}\mathbf{u}'}{u^2}=0, \quad (20)$$
$$\frac{a'}{a}+\frac{p'}{a}+2\left(1+\frac{p}{a}\right)\frac{\mathbf{u}\mathbf{u}'}{u^2}-3\left(1+\frac{p}{a}\right)\frac{\rho'}{\rho}=0. \quad (21)$$

Formulating equations (18) by k and u, with consideration of (19) we will obtain together with equations (20) and (21) a system of four uniform equations for the unknown $\frac{\mathbf{u}\mathbf{u}'}{u^2}; \frac{a'}{a}; \frac{p'}{p}; \frac{\rho'}{\rho}$. The characteristic determinant of this system is equalized to zero and we obtain equations which bind ω and k:

$$\omega_0^4-\omega_0^2[k^2u^2(1+2P)+(\mathbf{k}_0\mathbf{u})^2(2A+P)]+$$
$$+(\mathbf{k}_0\mathbf{u})^2[k^2u^2(3A+3P+6AP+5P^2)-(\mathbf{k}_0\mathbf{u})^2(3A^2+9AP+5P^2)]=0 \quad (22)$$

where

$$A=\frac{a}{\rho u^2}, \quad P=\frac{p}{\rho u^2}. \quad (22a)$$

If in this equation is written that $k = k_0$, it will designate phase expansion rates of small excitations with instantaneous vectors, which lie in the plane of separation:

$$V_{1,2}=v\nu\pm V_s; \quad V_{3,4}=v\nu\pm V_f, \quad (23)$$

where

$$V_s=\sqrt{\frac{D}{2}-\sqrt{\frac{D^2}{4}-F}}, \quad V_f=\sqrt{\frac{D}{2}+\sqrt{\frac{D^2}{4}-F}}, \quad (23a)$$

a $D=u^2[1+2P+\mu^2(2A+P)]$,

$F=u^4\mu^2[3A+3P+6AP+5P^2-\mu^2(3A^2+9AP+5P^2)]$.

It should be pointed out, that the velocities V_s and V_f - actually values, for which it is necessary that

$$F>0; \quad \frac{D^2}{4}-F>0. \quad (24)$$

Complex values V_s or V_f would indicate, that the plasma is unstable with respect to small excitations [2,6], irrespective of the tangential separation (rupture). From such considerations we will assume as fulfilled the conditions at which to the actual values k in equation (22) correspond actual values ω. From it under such conditions should arise the condition of positiveness of the free member in (22). Including (24) this gives:

$$3A+3P+6AP+5P^2>0. \qquad (25)$$

Instability can be caused only by complex values k_x (we assume that k_0-actual value).

We will take x - component of equation (18) and utilizing boundary conditions (10) which for solving (11), after inclusion of xi give:

$$\{p'+\rho uu'\}=0; \quad \{u'_x\,|\,(k_0u)\}=0, \qquad (25a)$$

we will obtain:

$$\left\{\frac{\rho}{k_x}[(k_0u)^2(1-A)-\omega_0^2]\right\}=0. \qquad (26)$$

From equation (22) we find k_x and substitude in (26)

$$\frac{\sqrt{1-\dfrac{(V-\nu_1v_1)^4-(V-\nu_1v_1)^2\mu_1^2u_1^2(2A_1+P_1)-\mu_1^4u_1^4(3A_1^2+9A_1P_1+5P_1^2)}{u_1^2[(V-\nu_1v_1)^2(1+2P_1)-\mu_1^2u_1^2(3A_1+3P_1+6A_1P_1+5P_1^2)}}}{\rho_1[\mu_1^2u_1^2(1-A_1)-(V-\nu_1v_1)^2]}=$$

$$=-\frac{\sqrt{1-\dfrac{(V-\nu_2v_2)^4-(V-\nu_2v_2)^2\mu_2^2u_2^2(2A_2+P_2)-\mu_2^4u_2^4(3A_2^2+9A_2P_2+5P_2^2)}{u_2^2[(V-\nu_2v_2)^2(1+2P_2)-\mu_2^2u_2^2(3A_2+3P_2+6A_2P_2+5P_2^2)}}}{\rho_2[\mu_2^2u_2^2(1-A_2)-(V-\nu_2v_2)^2]} \qquad (27)$$

where $V=\frac{\omega}{k_0}$. The signs k_{x1} and k_{x2} were selected so that the excitations remained finite at an increase in $|x|$, whereby it is also necessary to place a requirement, that the true parts of the radicals should be positive. At any values of parameters, equation (27) for V may have complex roots with positive imaginary part, which will indicate inconstancy of separation (rupture). Equation (27) can be written, by introducing speeds V_s and V_f as well as

$$V_A=\mu u\sqrt{1-A}, \quad V_m=\mu u\sqrt{\frac{3A+3P+6AP+5P^2}{1+2P}} \qquad (28)$$

(we like to point out that with respect to formulas (24) and (25) V_A and V_m - actual values) in form of

$$\frac{\sqrt{\dfrac{(V-\nu_1v_1-V_{s1})(V-\nu_1v_1+V_{s1})(V-\nu_1v_1-V_{f1})(V-\nu_1v_1+V_{f1})}{(V-\nu_1v_1-V_{m1})(V-\nu_1v_1+V_{m1})}}}{(V-\nu_1v_1-V_{A1})(V-\nu_1v_1+V_{A1})}=$$

$$=-d\frac{\sqrt{\dfrac{(V-\nu_2v_2-V_{s2})(V-\nu_2v_2+V_{s2})(V-\nu_2v_2-V_{f2})(V-\nu_2v_2+V_{f2})}{(V-\nu_2v_2-V_{m2})(V-\nu_2v_2+V_{m2})}}}{(V-\nu_2v_2-V_{A2})(V-\nu_2v_2+V_{A2})} \qquad (29).$$

where

$$d=\frac{\rho_1 u_1\sqrt{1+2P_1}}{\rho_2 u_2\sqrt{1+2P_2}}. \qquad (29a)$$

We will present (29) in a square and bring it into form of

$$\begin{aligned}&(V-\nu_1 v_1-V_{s1})(V-\nu_1 v_1+V_{s1})(V-\nu_1 v_1-V_{f1})(V-\nu_1 v_1+V_{f1})\times\\ &\times(V-\nu_2 v_2-V_{m2})(V-\nu_2 v_2+V_{m2})(V-\nu_2 v_2-V_{A2})^2(V-\nu_2 v_2+V_{A2})^2=\\ &=d^2(V-\nu_2 v_2-V_{s2})(V-\nu_2 v_2+V_{s2})(V-\nu_2 v_2-V_{f2})(V-\nu_2 v_2+V_{f2})\times\\ &\times(V-\nu_1 v_1-V_{m1})(V-\nu_1 v_1+V_{m1})(V-\nu_1 v_1-V_{A1})^2(V-\nu_1 v_1+V_{A1})^2.\end{aligned} \qquad (30)$$

Now is possible to express such adequate condition of tangential separation constancy.

We will arrange all speeds $\nu_1 v_1+V_{s1}$, $\nu_1 v_1-V_{s1}$... $\nu_2 v_2-V_{A2}$, $\nu_2 v_2+V_{s2}$, ... $\nu_1 v_1-V_{A1}$, which are included in (30) in the order of their increase. Since the speeds, included in tge left half of the equation, alternate with the velocities, included in the right half, and in addition they alternate and are "multiple" of velocities $\nu_2 v_2+V_{A2}$, $\nu_2 v_2-V_{A2}$ of the left half with the "multiple" velocities $\nu_1 v_1+V_{A1}$, $\nu_1 v_1-V_{A1}$ of the right half of the equation, then all roots of equation (30), and also equation (27), will be actual and the tangential separction (rupture) will be constant.

Next we will investigate equation (27) for the case when vectors v_1, v_2, u_1, u_2 are parlell, when $\nu_1=\nu_2=\mu_1=\mu_2\equiv\nu$.

For small values ν, imaginary roots V, if they do exist, tend toward zero with ν, consequently the second member under the radical equalling unity can be disregarded. We obtain equations

$$\rho_1[\nu^2 u_1^2(1-A_1)-(V-\nu v_1)^2]=-\rho_2[\nu^2 u_2^2(1-A_2)-(V-\nu v_2)^2], \qquad (31)$$

from which it is evident, that separation (rupture) will be constant upon fulfillment of condition

$$\rho_1 u_1^2(1-A_1)+\rho_2 u_2^2(1-A_2)-\frac{\rho_1\rho_2}{\rho_1+\rho_2}(v_2-v_1)^2>0. \qquad (32)$$

In case of arbitrary ν we will limit the class of separations by an additional requirement

$$\gamma_1=\gamma_2\equiv\gamma,\quad a_1=a_2\equiv a,\quad u_1=u_2\equiv u,\quad \rho_1=\rho_2\equiv\rho \qquad (33)$$

and will introduce a system of coordinates, in which

$$v_1 = -\frac{v_2 - v_1}{2} = -\frac{v_0}{2}, \qquad v_2 = \frac{v_2 - v_1}{2} = \frac{v_0}{2}. \tag{33a}$$

In this case it will be convenient to utilize equation (29), in which should be added

$$d = 1, \quad V_{s1} = V_{s2} \equiv V_s, \quad V_{f1} = V_{f2} \equiv V_f, \quad V_{m1} = V_{m2} \equiv V_m, \quad V_{A1} = V_{A2} \equiv V_A, \quad v_1 = -\frac{v_0}{2}, \quad v_2 = \frac{v_0}{2}. \tag{34}$$

When $\gamma = 0$ for separations (33) conditions (32) transform into condition

$$V_A > \frac{v_0}{2}. \tag{35}$$

In the other boundary case $\gamma = 1$ equation (29) (in which conditions (34) are included after after increasing into square, provided we eliminate the true roots $\omega = 0$ and $\omega = \pm\left(\frac{v_0}{2} \pm v_A\right)$, transforms into

$$V^4 - 2\left(V_s^2 + \frac{v_0^2}{4}\right)V^2 + \left(\frac{v_0^2}{4}\right)^2 - 2\left(\frac{v_0^2}{4}\right)V_s^2 + V_A^2\left(V_s^2 - V_m^2\right) + V_s^2 V_m^2 = 0, \tag{36}$$

where

$$V_s = u\sqrt{3(A+P)}, \quad V_m = u\sqrt{3(A+P) - \frac{P^2}{(1+2P)}}, \quad V_A = u\sqrt{1-\lambda} \tag{36a}$$

We will divide (36) into V_s^4 and introduce designations: $\frac{V_A^2}{V_s^2} = \alpha^2$, $\frac{V_m^2}{V_s^2} = 1 -$

$$\text{где } \gamma = \frac{P^2}{(1+2P)(3A+3P)}, \quad \frac{v_0^2}{4V_s^2} = \beta^2, \quad \frac{V^2}{V_s^2} = x^2, \text{ причому } \alpha, \beta, \gamma > 0. \tag{36b}$$

whereby

we will obtain

$$x^4 - 2x^2(1+\beta^2) + \beta^4 - 2\beta^2 + \gamma\alpha^2 + 1 - \gamma = 0. \tag{37}$$

Equation (37) has purely imaginary roots, because

$$(1-\beta^2)^2 - \gamma(1-\alpha^2) < 0, \tag{38}$$

and complex roots, because

$$4\beta^2 + \gamma(1-\alpha^2) < 0. \tag{39}$$

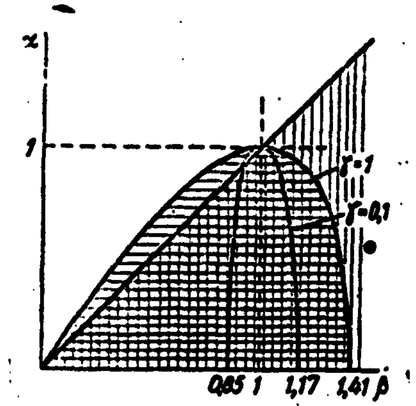
To the inequalities (38) and (39) should be added inequality

$$1 - \gamma > 0, \tag{40}$$

which is the result of inequality (25).

A direct check will convince, that the values alpha and beta, which satisfy inequality (39), should be eliminated by the fact, that the positiveness condition

of actual parts of radicals in (27) is not fulfilled here. In this way the inconstancy may originate only for alpha and beta values which do satisfy the inequality (38). On the drawing are shown zones of inconstant separation (hatched by horizontal lines) for two values gamma:0,1; 1. For gamma $\to$ 0 this zone is contracted into the section of straight beta = 1. On the drawing is drawn a straight line alpha = beta which separates the constancy zone from the inconstancy zone (hatched by vertical lines) of separation for ν = 0.



We shall now discuss intermediate values $0 < \nu < 1$. Excitations, corresponding to these values ν, can be considered as a superposition of two excitations, which correspond to values v = 0 and v = 1. Considering the fact that equation (10), (12) - (17) are linear, solutions for each one of the excitations will be indepebdent and in agreement with the already obtained for v = 0 and v = 1. Furthermore, figuring, that not the value of excitation amplitude, nor absolute values $\dot{k}_0$ affect the found zones of instability then for any arbitrary $0 < v < 1$ both zones exist together. For example, when gamma =1 the instability zone will be the entire hatched part of the area alpha and beta (see drawing).

In conclusion I want to express sincere thanks to prof. V.L. German for the work proposed by him.

Literature

1. G.F. Chew, M.L. Goldberger, F.S. Low, Proc. Roy. Soc. A. 236, 112, 1956 (its translation at PSF No. 7, 139, 1957).

2. R.V. Polovin, N.L. Tsintsadze, Ukrainskiy Fizicheskiy Zhurnal 4, 30, 1959

3. I.A. Akhiezer, R.V. Polovin, N.L. Tsintsadze, Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki, 37, 756, 1959.

4. S.I. Syrovatskiy, Transactions of Phys. Inst. 8, 15, 1956

5. L.D. Landau, Ye.M. Lifshits, Electrodynamics of Solid Media, GITTL, Moscow, 1957

6. R.Z. Sagdayev, collection of rep. Physics of Plasma and Problems of Controlled Thermonuclear Reactions " vol. 3, Izdat. Akad. Nauk. SSSR, Moscow 1958. p. 268

7. A.G. Kurosh, Course in Higher Algebra OGIZ, Gostekhizdat, 1946.

The O.M. Gor'kiy State Univ. at Khar'kov

DISTRIBUTION LIST

DEPARTMENT OF DEFENSE	Nr. Copies
HEADQUARTERS USAF	
AFCIN-3D2	1
ARL (ARB)	1
OTHER AGENCIES	
CIA	1
NSA	6
DIA	9
AID	2
OTS	2
AEC	2
PWS	1
NASA	1
ARMY (FSTC)	3
NAVY	3
NAFEC	1
RAND	1
AFCRL (CRXLR)	1

MAJOR AIR COMMANDS	Nr. Copies
AFSC	
SCFDD	1
DDC	25
TDBTL	5
TDBDP	2
AEDC (AEY)	1
SSD (SSF)	2
AFWL (WLF)	1
ASD (ASYIM)	2
ESD (ESY)	1